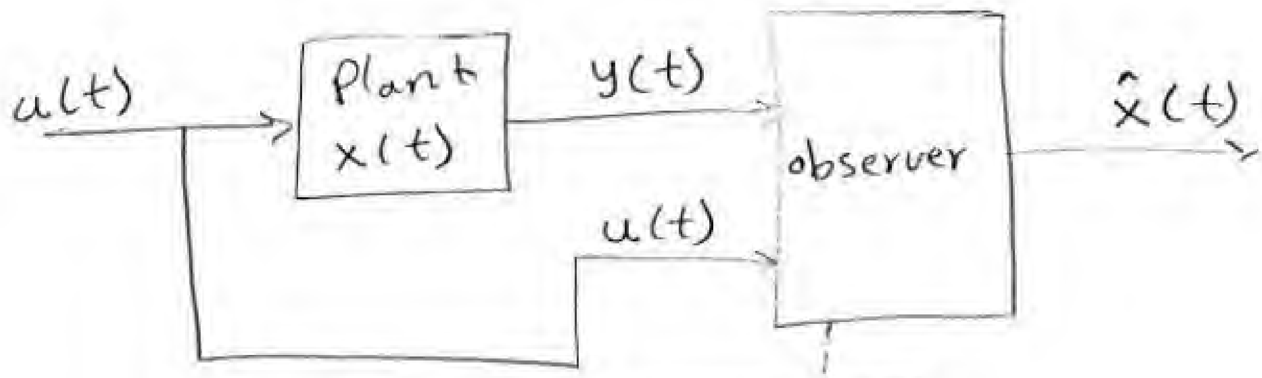
Control Lec 11

observer design

* The observer is used to estimate the unmeasurable states.

observer

- Full order (estimation is done for all states)
- reduced order (estimation is done for some states not for all states)



$$\boxed{e(t) = x(t) - \hat{x}(t)}$$

* The observer eqn

$$\dot{\hat{x}}(t) = (A - Gc)\,\hat{x}(t) + B\,u(t) + G\,y(t)$$

* The design is to find gain matrix "G"

ـــ الـ specs الخاصة بالـ (design) هنا مرتبطة

بـ (G) .

* To obtain the gain G, the design specs. will be contained the required transient response for the observer (given desired poles)

$\boxed{1}$ desired ch. equation $\alpha_0(s)$

$$\alpha_o(s) = (s-P_1)(s-P_2)\ldots \quad (1)$$

$P_1, P_2 \rightarrow$ desired poles (given)

$\rightarrow$ desired ch. equation in terms of G

$$|sI - (A - GC)| = 0 \quad (2)$$

$\boxed{2}$ $G = \alpha_o(A)\, M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

$M_o \Rightarrow$ observability Matrix

$$\alpha_o(A) = \alpha_o(s)\Big|_{s \rightarrow A}$$

$\boxed{Ex}$ Design an observer for the following system:-

$$\dot{x} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & -1 \end{pmatrix} x(t)$$

The observer poles locate at $-1, -2$

<u>sol</u>

1) <u>desired ch-equation for the observer:-</u>

$$\alpha_0(s) = (s+1)(s+2) = 0$$

$$\boxed{\alpha_0(s) = s^2 + 3s + 2}$$

$$\alpha_0(A) = A^2 + 3A + 2I$$

$$\alpha_0(A) = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + 3 \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\alpha_0(A) = \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix}$$

2) $M_0 = \begin{pmatrix} C \\ CA \end{pmatrix}$

$$CA = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$M_0^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$G = \alpha_0(A) M_0^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{G = \begin{pmatrix} 12 \\ 7 \end{pmatrix}}$$